## Curriculum topics:

- Geometric Shapes
- Patterns \&

Relationships

- Star Polygons
- Modular Multiplication
- Prime \&

Composite
Numbers

- Native American Culture


## Subjects:

Math, Art,
Social Studies

Grade range: 6-9

Who we are:
Resource Area for Teaching (RAFT) helps educators transform the learning experience through affordable "hands-on" activities that engage students and inspire the joy and discovery of learning.

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# ADVANCED MATHEMATICAL DREAM CATCHERS 

Explore Mathematical Properties, Star Polygons Modular Arithmetic, \& More!
ive American Indian culture and mathematical concepts combine to make dream catchers. This activity leads to higher mathematical investigations, including properties of circles and modular star polygons.


## Materials required

Per person:

- Hoop, 15 to 25 cm ( 6 " to 10 ") in diameter
- Mini binder clips, 8
- Pony beads, 1 each of 5 different colors
- Small container for beads
- String, 3 different colors, ~1 m (3 ft) each
- Feathers, multi-colored, 3 to 5
- Protractor
- Geometric compass

Note: If the hoop contains a screw on top, tie a string at the base of the screw (to hold the hoop together). Optional: Tie a feather onto the screw.
See illustrations to the right.


## To do and notice

1 To mark 6 equally spaced points around the rim of the hoop:

- Use a protractor to locate a central angle with measure equal to $360^{\circ}$ divided by $\boldsymbol{n}$, where $\boldsymbol{n}=6$, the number of equally spaced points to be marked.
- Use a compass to find the arc length associated with this angle and mark this
 length 6 times along the hoop.
- Label the points clockwise from 0 to 5 along both the top and the bottom rim of the hoop.

2
Attach a binder clip over each marked point on the top rim of the hoop. Fold down the inner binder clip loops so the loops point to the center of the hoop.

3
Slip the end of a string through the loop of the first clip (at point 0 ) and tie a knot. Pass the other end of the string through a loop on the binder clip at point 1 and then at point 2. Continue in sequential order, counting one clip each time until returning to point 0 . At any time, add beads to the string. Cut and tie the string. Identify the polygon. Because there is
 one count between any two connected clips ( $0 \& 1 ; 1 \& 2$, etc.) $r=1$. So for this polygon $\mathbf{n}=6$ and $\mathbf{r}=\mathbf{1}$.

Using a different colored string, starting at point 0, count 2 clips and connect to point 2; then count 2 clips and connect to point 4; count 2 clips and connect to point 0 . Tie a knot and cut string. Note the sequence of connected points is: point 0 to point 2, point 2 to point 4, and point 4 back to point 0 , written as: 0-2-4-0.


Since not all the points are connected, use the same color string to connect the remaining points, starting at point 1 and then connecting to point 3, then point 5 and back to point 1. (Sequence is $1-3-5-1$ ) See figure above right. Tie a knot and cut string. Which polygons are formed? ( $\mathbf{n}=6$ and $\mathbf{r}=\mathbf{2}$ )

Using the $3^{\text {rd }}$ color string, start at point 0 , count 3 clips and connect to point 3. Tie a knot and cut the string. Using the same color, connect point 1 to point 4 (counting 3 clips), tie a knot and cut. Similarly connect point 2 to point 5. The three sequences are 0-3; 1-4; and 2-5. (Note: sequences $3-0,4-1$, and $5-2$ will create the same string pattern). ( $\mathbf{n}=6$ and $\mathbf{r}=3$ )

6 Tie a string loop to the top of the dream catcher to hang it. Option: add feathers (see illustrations on page 1).


If $\boldsymbol{n}$ and $\boldsymbol{r}$ have no common factors other than 1 ( $\boldsymbol{n}$ and $\boldsymbol{r}$ are relatively prime numbers) then all $\boldsymbol{n}$ points on the hoop are connected by the time the string returns to the starting point, as in step 3. The resulting design is called a regular star polygon! Star polygons are represented by the notation: $\{\boldsymbol{n} \mid \boldsymbol{r}\}$, meaning: the circle is divided by $\boldsymbol{n}$ equally spaced points, where every $\boldsymbol{r}^{\text {th }}$ point is connected. The star polygon in step 3 is $\{6 \mid 1\}$.

8 If $\boldsymbol{n}$ and $\boldsymbol{r}$ have common factors other than 1 , then not all points will be connected with one string. Start again at an unconnected point to complete the connection of all $n$ points in the sequence. The resulting design is called a modified star polygon! The modified star polygons $\{6 \mid 2\}$ and $\{6 \mid 3\}$ are shown in steps 4 and 5 above.

## The content behind the activity

Creating dream catchers is an excellent way to explore mathematics. Have students look for simple geometric shapes within the webbing; find relationships between the number of connected points and the total number of points on the circle (whether they are relatively prime or relatively non-prime); and find similar shapes and lines of symmetry. Students can explore modular arithmetic; whether there are any functional relationships between the shapes and other properties of the circle. Does changing the stringing pattern from one loop to the next, affect the number, size, or types of shapes?

## Relatively prime numbers

Two integers are said to be "relatively prime" if the largest divisor they have in common is 1 . For example: 4 and 12 are not relatively prime because 2 divides evenly into both numbers. The numbers 35 and 27 are relatively prime since no number larger than 1 goes evenly into both.

## Historical background information

This activity weaves beautiful Native American Indian tradition with mathematics.
The first dream catchers were crafted by the Ojibwe (Chippewa) tribe. Legend tells of a "spider woman" named Asibikaashi whose magical web had the power to trap the Sun. Dream catchers were traditionally hung above sleeping babies to catch bad dreams and let only good dreams pass through to the child. Later, the bad dreams would disappear when the first rays of sunlight struck the web.

When the native Ojibwe nation dispersed to the four corners of North America, Asibikaashi found it hard to share her webs with everyone who wanted one. So, mothers, sisters, and grandmothers started creating their own webs using flexible hoops made from willow branches.

Traditional dream catchers have 8 holes along the outer rim to represent a spider's 8 legs. Today, many different Native American Indian tribes make dream catchers in a wide variety of styles.

## Curriculum

## Standards:

Ratios
(Common Core Math Standards: Grade 6, Ratio \& Proportions, 1)

## Pi

(Common Core Math Standards: Grade 6, Number System, 6)

Area and Circumference of a circle
(Common Core Math Standards: Grade 6, Geometry, 6)

Problems involving angles \& area (Common Core Math Standards: Geometry, Grade 6, 1; Grade 7, 4, 5, \& 6)

Problem Solving and
Reasoning
(Common Core Math
Standards:
Mathematical Practices
Grades 6-9)
Creative art, materials, and making (National Visual Arts Standards: Creating: Grades 6-9,
1.1; Grades 6-9, 2.1)

Art, society, and culture (National Visual Arts Standards: Connecting:
Grades 6-9, 11.1)

Traditions \& culture
(National Curriculum for Social Studies:
Theme 1, Culture)

## Learn more

- For more math activities using regular and modified star polygons and modular multiplication, visit: www.raft.net/raft-idea?isid=682

Related activities: See RAFT Idea Sheets:

## Mathematical Dream Catchers - <br> http://www.raft.net/ideas/Mathematical Dream Catchers.pdf

Patterning with Polygons http://www.raft.net/ideas/Patterning with Polygons.pdf

## Scalloped Circle String Art -

http://www.raft.net/ideas/Scalloped Circle String Art.pdf
What Shapes Can It Become -
http://www.raft.net/ideas/What Shapes Can It Become.pdf


## Resources

Visit www.raft.net/raft-idea?isid=682 for "how-to" video demos \& more ideas!
See these websites for more information on the following topics:

- Legend of the Native American Indian Dream Catcher -
http://www.firstpeople.us/FP-Html-
Legends/TheLegendOfTheDreamcatcher-Chippewa.html
- Mathematical Dream Catcher Mandalas -
http://clem.mscd.edu/~talmanl/Mandalas.html
- Modular Arithmetic http://mathforum.org/library/drmath/view/62930.html
- Regular Star Polygons -
http://mathworld.wolfram.com/StarPolygon.html
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