RAFT IDEAS

Topics: Conditional Probability

Materials List
(per pair of players)
- ✓ Cups, 3 – labeled A, B, and C
- ✓ Small object ("prize"), 1
- ✓ The Monty Hall Tally Sheet, 1 per game

This activity can be used to teach:
Common Core Math Standards:
- Statistics and Data (Statistics and Probability, Grade 6, 1-5; Grade 7, 5-6),
- Probability of events (Grade 7, Statistics and Probability, 7-8),
- Independent and conditional probability (High School, Statistics and Probability- Conditional Probability 2, 3, 6)
- Problem Solving and Reasoning (Mathematical Practices Grades 6-12)

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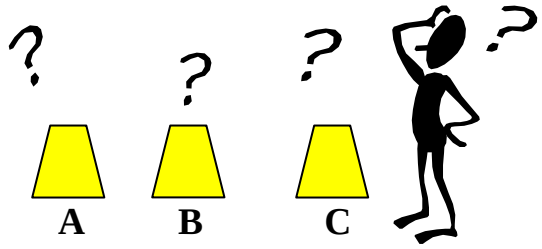
Monty Hall Makes a Deal!

Exploring Conditional Probability!



Play a game show with only one choice out of three to win a prize! Learn to improve the chances of getting the prize.

Playing the Game (for 2 players)

Round One - "Keep Initial Choice"

1. Decide on a person for each of the following positions:
   a. The "Game Host" always knows which cup the prize is hidden under.
   b. Monty's "Guest": is the other player. The Guest does not know which cup contains a prize.
2. Without showing the other player, the Game Host tucks a hidden "prize" under one of the three cups.
3. The Guest begins the first trial by choosing a cup. Before revealing the Guest's choice, the Game Host lifts one of the other two cups (the one not containing a prize) and shows its empty contents to the Guest.
4. The Game Host offers the Guest an opportunity to switch their initial cup choice with the other unrevealed cup. In this round, the Guest **always keeps** their initial choice.
5. The Game Host reveals the contents of the Guest's cup. If the Guest's cup contains the prize, the Guest records a check on the tally sheet for that trial under the column for Round One.
6. Repeat steps 2 through 5 above for 19 more trials.
7. Calculate the number of times the prize was found in this round. Divide this number by 20 (the total number of trials). The resulting number is the "Experimental Probability" of getting a prize when the Guest always stays with the initial choice. Compare that number to the Calculated Probability (= 1/3) of these events happening – see the **Math Behind the Activity** for the explanation.

Round Two "Switch Initial Choice"

8. Players switch roles. Play as in Round One, **except** in this round the Guest **always switches** cups when offered a chance to do so.
9. The Calculated Probability for Round Two is 2/3 – see the **Math Behind the Activity** for the explanation.

Analyzing the Results

10. Compare the results for the experimental and computed probabilities for Round One and Round Two.
11. Did the experimental probabilities come close to the calculated conditional probabilities? Why or why not?
12. Compile the results from many games -look at the results from a whole class or play the game a number of times. How do the experimental and calculated probabilities compare?

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The Math Behind the Activity

Monty Hall Makes a Deal! is named after Monty Hall who was the original Game Host of the American television game show *Let's Make a Deal*. In that show, contestants had to choose what was behind one of three hidden doors and had a chance to change their answer after seeing what was behind one of the un-chosen doors.

There are no guarantees for any given turn, but if this game is played many times, switching an initial choice is the best strategy! Why? Because the conditional probability of an event is the probability that the event will occur given the knowledge that another event has already occurred. In the Monty Hall problem, once one of the empty cups is eliminated and players see the 2 remaining cups, they usually think there's a 1 in 2 chance that they have picked correctly. But, in terms of probability, their first choice has a 1 in 3 chance of being correct. After the first cup is eliminated, if a player switches from their first choice, the chances of winning increase to 2 in 3.

Take at look at the following scenarios:

If the Guest chooses A and does not switch

Cup A	Cup B	Cup C	Result
Prize	Nothing	~~Nothing~~	WIN!
Nothing	Prize	~~Nothing~~	LOSE
Nothing	~~Nothing~~	Prize	LOSE

If Guest chooses A and then switches

Cup A	Cup B	Cup C	Result
Prize	Nothing	~~Nothing~~	LOSE
Nothing	Prize	~~Nothing~~	WIN!
Nothing	~~Nothing~~	Prize	WIN!

The left-hand chart shows an example of where the guest has chosen Cup A and does not switch. Crossed out boxes on this chart indicate cups that have been eliminated (shown to be empty) by the Game Host. There is one "win" and two "lose" results. If charts are made for cups B and C, the same results would be shown, thus there is a 1-in-3 chance of winning the prize by not switching.

The right-hand chart shows an example of where the guest has chosen Cup A and then switches cups when offered a chance to do so. There are two "wins" and one "lose" result. Thus if the guest switches each time after one of the empty cups is turned over, they have a 2-in-3 chance of winning. Again, if this chart was made for cups B and C, the same results would be shown, meaning that this guest will win 2 out of 3 times!

Taking it Further

- Use four cups with one hidden prize. Let the guest choose the first cup, but before it is revealed, the Host shows them the contents of two of the three remaining cups before the final selection is made. Does this change the condition?
- Explore other conditional probability scenarios such as: what is the probability of two dice summing to 10, given that both rolled dice values are odd?

Web Resources (Visit www.raft.net/raft-idea?isid=657 for more resources!)

- Alternative examples of the Monty Hall Dilemma - http://www.cut-the-knot.org/hall.shtml
- More conditional probability examples - http://www.mathgoodies.com/lessons/vol6/conditional.html
- Teacher designed math courses from the New Jersey Center for Teaching & Learning – https://njctl.org/courses/math

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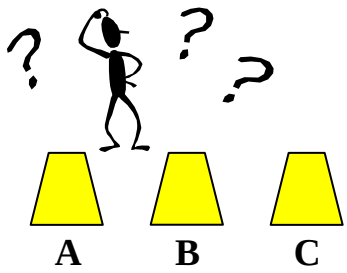
The Monty Hall Tally Sheet

Trial Number	Round 1: Guest **Kept** Initial Choice & Won	Round 2: Guest **Switched** Initial Choice & Won
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
Total number of wins		
	Compare the **Experimental Probability** (number of wins/number of trials) to the **Calculated Probability** (in this scenario = one out of three = **1/3**) of choosing a winning cup.	Compare the **Experimental Probability** (number of wins/number of trials) to the **Calculated Probability** (in this scenario = two out of three = **2/3**) of choosing a winning cup.

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