

Curriculum topics:

- Patterns
- Variables
- Functions
- Algebra
- Geometry
- Geometric Patterns Described Algebraically

Subject:

Mathematics

Grade range: 6 – 8

Who we are:

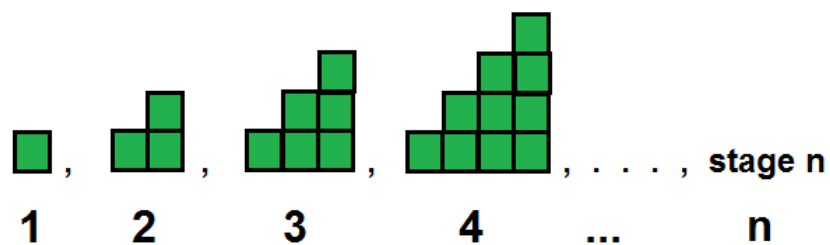
Resource Area for Teaching (RAFT) helps educators transform the learning experience through affordable “hands-on” activities that engage students and inspire the joy and discovery of learning.

For more ideas and to see RAFT Locations

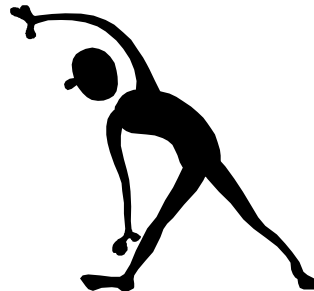
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SHAPE UP WITH ALGEBRA

Discover an algebraic function based on successive patterns



Using centimeter cubes, build successive shapes based on a geometric pattern. Compare the shapes at successive stages to find an algebraic functional relationship.



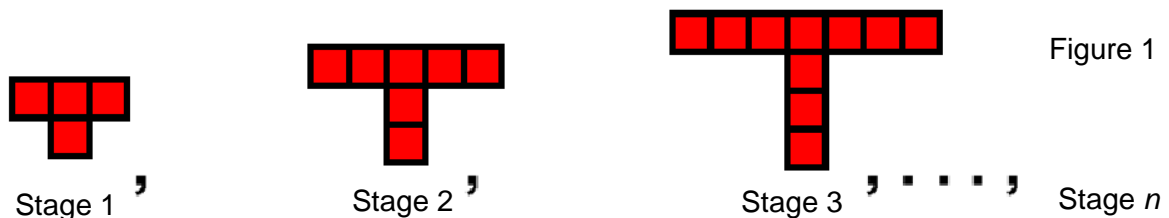
Materials required

(for teams of 2 – 4 students)

- Small cm cube squares, 100 of one color
- **Shape Up with Algebra** Recording Sheet, 1 per team, page 5

To do and notice

- 1 Place the cm cubes into one pile in front of each team.
- 2 Have each team use centimeter cubes to arrange the first three “T” patterns in each successive stage of the sequence. See figure 1 below. For another example, see figure 2 on page 3.



- 3 Review functions and T-tables with the students. For an example of a T-table, see figure 3.
- 4 Ask the teams to count the number of cm cubes for each of the first three stages, and to enter this information under the “Number of Cubes” column on the **Shape Up with Algebra** Recording sheet.
- 5 Ask the teams to write whatever pattern they see at each stage under the “Describe the Pattern that you see at each stage” column on the **Shape Up with Algebra** Recording sheet.
- 6 Have students build the 4th and 5th stages with cm cubes and record their findings as before. Does the same relationship continue? Ask teams to predict the number of cm cubes needed for the 6th stage, and then have them build the 6th stage and record the findings.
- 7 Suggest to teams that if “ n ” is the stage number in this sequence, how can they predict the number of cm cubes needed when $n = 7$? Then when $n = 10$?, and when $n = 100$? Ask the teams to predict the number of cm cubes needed to build any stage n ?
- 8 Ask teams to describe the changes in the number of cm cubes at each stage (e.g., one more cube is added to the end of each “arm” and to the “base” of the “T” at every stage).

9

Ask teams to write an algebraic expression that describes the number sequence (e.g., the function) for the n^{th} stage.

10

Ask teams to reflect on what was learned. Then have the teams describe why knowing an algebraic function for any stage n is more helpful than just continuing to build more and more stages with cm cubes.

The math behind the activity

This activity gives students a hands-on approach in exploring geometric patterns and in discovering how such models can be translated into algebraic functions. In order to truly appreciate algebra, students need to recognize relationships between patterns, expressions, equations, variables, and functions. Students need to move from the concrete (building a sequence of patterns in stages) to the abstract (representing the pattern for any stage as an algebraic function.)

By examining patterns and structures in several different representations of a math problem, 1) a concrete representation, 2) a table, 3) a verbal description, and 4) an algebraic formula, students detect regularities, formulate generalizations, and make conjectures. Practicing these mathematical reasoning and proof techniques helps students apply and adapt a variety of strategies to approach and solve many types of math problems.

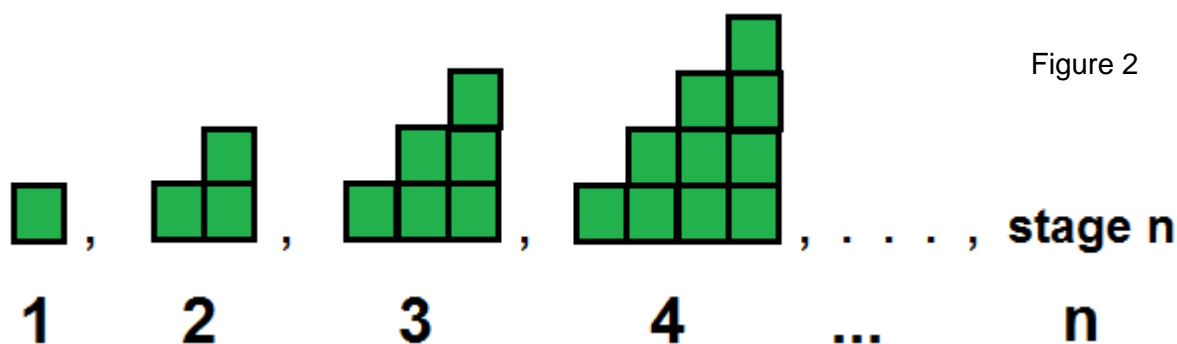


Figure 2

n	$f(n)$
1	1
2	3
3	6
4	10
\vdots	\vdots
n	$n(n + 1) \div 2$

Figure 3

Curriculum Standards:

Expressions and Variables
(Common Core Math Standards: Grade 6, Expressions and Equations, 2, 6, & 9)

Solving numerical and algebraic problems
(Common Core Math Standards: Grade 7, Equations and Expressions, 4)

Solving area, volume, and surface area problems
(Common Core Math Standards: Grade 7, Geometry, 6)

Graphs and equations for proportional relationships
(Common Core Math Standards: Grade 8, Expressions and Equations, 5)

Functions as rules and graphs
(Common Core Math Standards: Grade 8, Functions, 1)

Properties of functions
(Common Core Math Standards: Grade 8, Functions, 2 & 5)

Problem Solving and Reasoning
(Common Core Math Standards: Mathematical Practices Grades 6-8)

Additional standards at:
<http://www.raft.net/raft-idea?isid=653>

Learn more

- Find the area and perimeter at each stage of the sequence and determine each function.
- Graph the *stage number* vs. the *number of cm cubes* at each stage. If the discrete points are connected, would the slope of the connecting line indicate anything about the relationship of the number of cubes from stage to stage?
- Use centimeter cubes, coins, tooth picks, and other items to make and explore different sequential patterns.
- Use two or more different colored objects in a pattern. Build a sequence and look for a function for each of the colored “variables” in the shapes. See example below & visit www.raft.net/raft-idea?isid=653 for more patterns.



Related activities: See RAFT Idea Sheets:

Algebra Rummy -

<http://www.raft.net/ideas/Algebraic Rummy.pdf>

Aquatic Quadratics -

<http://www.raft.net/ideas/Aquatic Quadratics.pdf>

Dive Into Square Pools -

<http://www.raft.net/ideas/Dive into Square Pools.pdf>

Meet My Function Machine!-

<http://www.raft.net/ideas/Meet My Function Machine.pdf>

Modeling Simple Equations -

<http://www.raft.net/ideas/Modeling Simple Equations.pdf>

Occasions for an Equation! -

<http://www.raft.net/ideas/Occasions for an Equation.pdf>

Slippery Slopes -

<http://www.raft.net/ideas/Slippery Slopes.pdf>

Resources

Visit www.raft.net/raft-idea?isid=653 for “how-to” video demos & more ideas!
See these websites for more information on the following topics:

- Reasons for understanding patterns & algebra** -
<http://teachingtoday.glencoe.com/howtoarticles/preparing-middle-school-students-for-algebra>
- Ideas for teaching about patterns & algebra** -
http://www.curriculumsupport.education.nsw.gov.au/secondary/mathematics/years7_10/teaching/algebra.htm
- More interactive patterning activities** -
<http://www.shodor.org/interactivate/activities/>
- Khan Academy resources on functions** -
<https://www.khanacademy.org/math/algebra/algebra-functions>
- Teacher designed math courses from the New Jersey Center for Teaching & Learning** - <https://njctl.org/courses/math>

Acknowledgements:

Algebra in the Concrete, by Mary Laycock and Reuben A. Schadler.
(Revised Edition 1987) Published by Activities Resource Company, Inc.

Shape Up with Algebra Recording Sheet!

Stage Number	Number of Cubes	Describe the Pattern that you see at each stage
1		
2		
3		
4		
5		
6		
7		
. . . ,		
10		
11		
. . . ,		
n		