Curriculum topics:

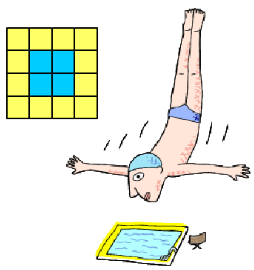
- Patterns
- Variables
- Functions
- Graphing
- Algebra

Subject: Mathematics

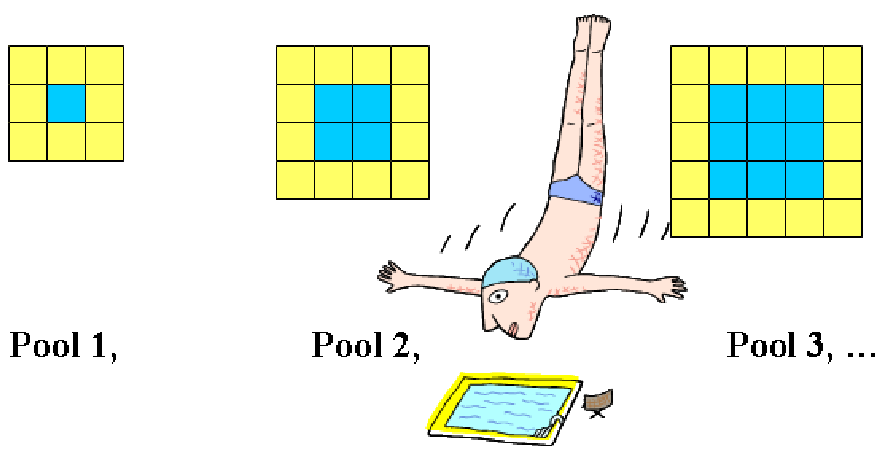
Grade range: 6 – 12

DIVE INTO SQUARE POOLS

Make a splash tiling borders around square swimming pools!



"Build" increasingly larger square pools with a single tile border to look for patterns in the number of tiles needed. Develop a general tile formula for any size square pool.



Who we are:

Resource Area for Teaching (RAFT) helps educators transform the learning experience through affordable "hands-on" activities that engage students and inspire the joy and discovery of learning.

For more ideas and to see RAFT Locations

www.raft.net/visit-raft-locations

Materials required
(for teams of 2 – 4 students)

- Small cm cube squares,
  100 blue colored, 100 of another color
- Graph paper
- Scratch paper
- Optional: pencils or markers, colors the same as the cm cubes

To do and notice

1 Place the cm cubes into two piles by color in front of each team.

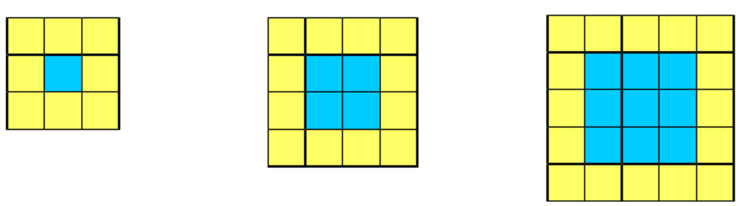
Teaching Tip: Review functions, T-tables, domain, and range with the students prior to engaging in the activity.

2 Provide each team player with one sheet of graph paper, one color of each pencil or marker, and scratch paper.

3 Relate the following to the students:

"You have been given the job of tiling square swimming pools. Each pool has a water surface measured in blue square centimeter cubes (call them the *'pool tiles')*. Surrounding each square pool is a deck border made of the other colored cm cubes (call them the *'deck tiles*'). The first pool has only one pool tile, and the second pool needs enough pool tiles to make the water surface into a square made from cm cubes.

You need to figure out how many pool tiles and how many deck tiles are needed to build any size square pool. Look for a relationship between *each pool's number* to *the number of pool tiles and deck tiles* it has, as you first build the smallest square pool and then move on to build increasingly larger square pools."



Pool 1, **Pool 2,** **Pool 3, …**

4 Suggest the teams begin building different sized square pools with the cm cubes and exploring any relationships they find in the *number of pool tiles and deck tiles* used for each *pool number.* Then suggest students start building pools sequentially from the first pool, to the second pool, and so forth. On graph paper have students draw each pool, using one square on the paper to represent each pool and deck tile. See figure 1. Option: color the squares to distinguish between the deck and pool tiles.

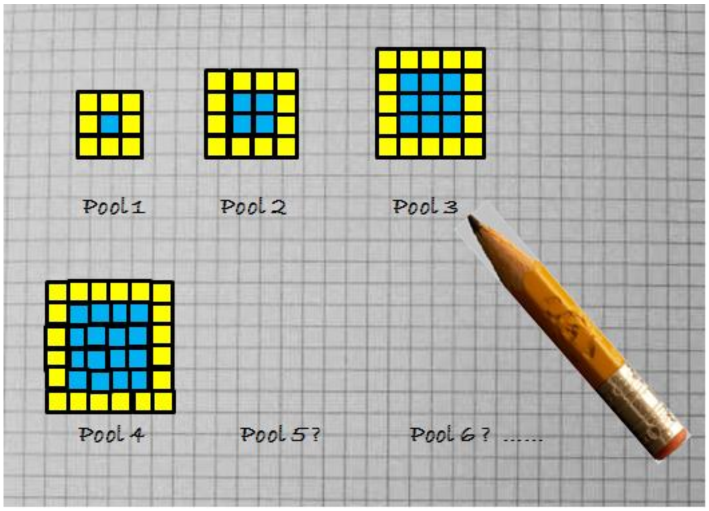


Figure 1

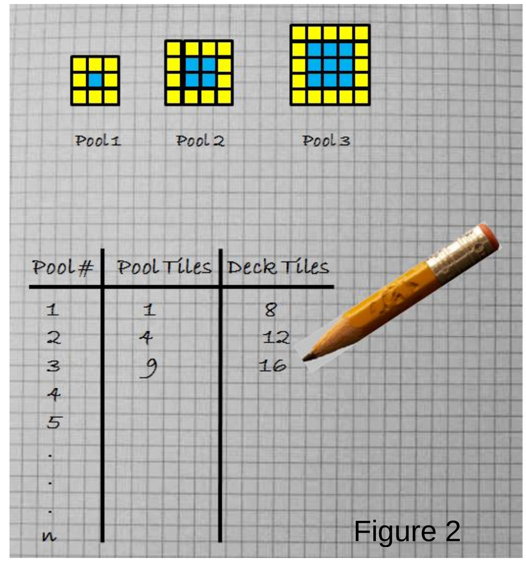
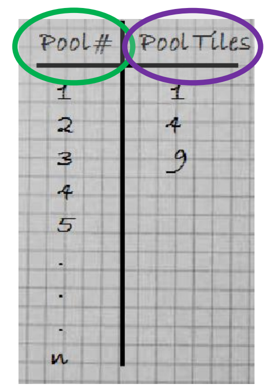


Figure 2

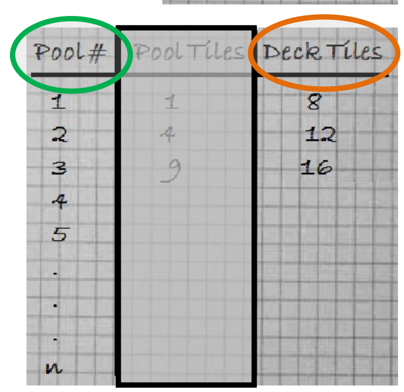
5 Have students transfer their information onto a "T" table listing *the number of each pool,* its *number of pool tiles,* and *its number of deck tiles.* See figure 2.

6 Have students find a relationship between ***a pool's number*** and the ***number of pool tiles*** it has.



- How does the number of pool tiles change from one pool to the next?
- How many pool tiles do they predict the **5th** pool will have? The **tenth** pool? The **one-hundredth** pool?
- Have students discover a "rule" for finding the number of pool tiles for any square pool (e.g., the function for the **nth** pool).

7 Have students create larger pools with the cm cubes and keep adding the information to their T table. Ask them to look for a mathematical relationship between ***the pool number*** and the ***number of deck tiles***.



- How does the number of deck tiles change sequentially from pool to pool? Have students describe how this relationship was discovered.
- Ask students to find the number of deck tiles for the **100th** pool, and eventually for any sized pool (e.g., the function for the **nth** pool).

8 Exploration questions: If there are 144 pool tiles, what is the length of one deck side of the pool? How many deck tiles are needed for that pool? Could there be a square pool with 4,000 deck tiles? How many tiles are in a pool with "n" tiles on one edge of the pool (not counting the deck)?

The math behind the activity

In the middle grades, algebraic thinking moves from informal explorations to formal algebraic considerations; from the concrete to the abstract. The goal of this activity is for students to discover interconnectedness between visual patterns, methods of organizing information (e.g., through the use of T-tables, functions, and graphs), algebraic and geometric symbolism and formulas for problem solving.

Curriculum Standards:

Expressions and Variables (Common Core Math Standards: Grade 6, Expressions and Equations, 2, 6, & 9)

Solving numerical and algebraic problems (Common Core Math Standards: Grade 7, Equations and Expressions, 4)

Solving area, volume, and surface area problems (Common Core Math Standards: Grade 7, Geometry, 6)

Functions as rules & graphs; Properties of Functions (Common Core Math Standards: Grade 8, Functions, 1, 2, & 5)

Problem Solving and Reasoning (Common Core Math Standards: Mathematical Practices Grades 6-12)

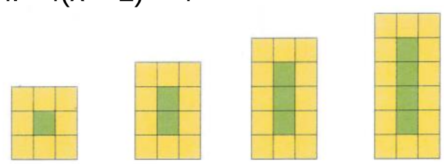
Additional standards at: http://www.raft.net/raft-idea?isid=614

Learn more

- On a coordinate grid, graph the number of pool tiles along the y axis versus the pool number along the x axis using a blue pen. Then, use a pen or marker the same color as the deck tiles to graph the number of deck tiles along the y axis versus the number of the pool along the x axis. Connect the coordinates for the pool numbers vs. pool tiles. Connect the coordinates for the pool numbers vs. the deck tiles. What happens after the 5th pool? What does the slope of each graph tell you? What does the y-intercept indicate along each graph?
- If the function for deck tiles were $3n + 4$, could you show a picture for each pool stage?
- What type of graph is made by the coordinates for the border tiles in the side of a pool & the number of bottom tiles?
- Discuss the thoughts behind the following solutions for the number of border tiles in a square pool that has a bottom that measures x by x:

  a. $1 + x + 1 + x + 1 + x + 1 + x$
  b. $4(x + 1)$
  c. $x + x + x + x + 4$
  d. $(x + 2)^2 – x^2$
  e. $4(x – 1) + 8$
  f. $2(x + 2) + 2x$
  g. $2[x + 2(x ÷ 2) + 2]$
  h. $4(x + 2) – 4$

- Perform similar operations with other pattern shapes (e.g., rectangular pools). Try the Garden Borders pattern to the right.



Related activities: See RAFT Idea Sheets:

Aquatic Quadratics – http://www.raft.net/ideas/Aquatic Quadratics.pdf
Meet My Function Machine!- http://www.raft.net/ideas/Meet My Function Machine.pdf
Modeling Simple Equations – http://www.raft.net/ideas/Modeling Simple Equations.pdf
Occasions for an Equation! – http://www.raft.net/ideas/Occasions for an Equation.pdf
Shape Up with Algebra - http://www.raft.net/ideas/Shape Up with Algebra.pdf

Resources

Visit www.raft.net/raft-idea?isid=614 for “how-to” video demos & more ideas!

See these websites for more information on the following topics:

- **Rectangular pool tiling problem** - www.teachersnetwork.org/ntny/lessonplans/takhui.htm
- **More patterning problems** - www.learner.org/courses/learningmath/algebra/overview/lma3_5.pdf
- **Khan Academy resources on functions** - https://www.khanacademy.org/math/algebra/algebra-functions
- **Teacher designed math courses from the New Jersey Center for Teaching & Learning –** https://njctl.org/courses/math

Acknowledgements:

Experiences with Patterning, by Joan Ferrini-Mundy, Glenda Lappan, and Elizabeth Phillips, in *Teaching Children Mathematics* (February 1997), pp. 282-288. UCLA Math Programs for Teachers/LUCIMATH Project.