Curriculum topics:

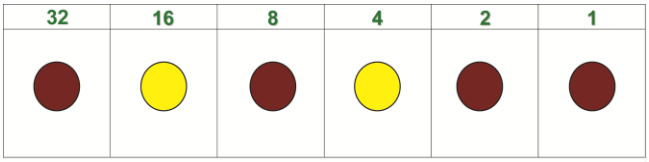
- Binary (base-2)
- Number Bases
- Place Value
- Exponents
- Factors

Subject: Math

Grade range: 4 – 12

BINARY DOTS

Seeing base-2 spots



$$43 = 2^5 + 2^3 + 2^1 + 2^0 = 101011_2$$

Counting in binary is quite straightforward and makes for an interesting and compelling activity for any student who can do simple addition. This uncomplicated game creates an opportunity to practice counting in binary. It reinforces understanding of place value and can be extended into a discussion of other bases and number systems.

32	16	8	4	2	1

Figure 1

Materials required

(if using the RAFT Kit)

- Game Board, 6x2 grid
- 63 preprinted playing cards, numbers 1 – 63
- Circle tokens (red and yellow)

(if creating your own set)

- Poster board or paper
- Blank playing cards or index cards
- Permanent marker
- Circle tokens (6 in each of two colors)

How to build it

If using the RAFT Kit, go to "Playing the game" below.

1 Create the game board by marking a 6x2 grid on a suitable piece of poster board or paper (see Figure 1 on page 1). Write in the place values 32, 16, 8, 4, 2, and 1 as shown in Figure 1. These place values belong to the decimal number system (the numerical base we use widely today), also called base-10. While these numbers could be written as $32_{10,}$ $16_{10,}$ $8_{10,}$ $4_{10,}$ $2_{10,}$ $1_{10,}$ we usually leave off the subscript.

2 Create cards that have numbers between 1 and 63 written on them. The number 63 is the highest base-10 number that can be created with this game board. (Note: the total number of cards is flexible, no need to have a card for each number.)

3 Assign the number "1" to one color of token, and "0" to the other. In the instructions below, red = 1 and yellow = 0. **Optional:** Use a marker to write ones and zeros on the circle tokens.

Playing the game

1 To play the game, draw a base-10 number card and change it into binary using the tokens. Decide which of the place values must be "on" (red = 1 = "on") and which must be "off" (yellow = 0 = "off") in order to represent the same number in binary.

For example, the number "43" is drawn. The sum total of the "on" numbers will add up to 43. The only combination that sums to 43 is (32 + 8 + 2 + 1). Place the red tokens in the spaces under the numbers that should be "on," and the yellow tokens in the spaces under the numbers that should be "off" (see image at the top of page 1).

2 Write down the binary number on a piece of paper placed underneath the grid by writing "1" underneath each "on" number and a "0" underneath each "off" number. (e.g.: 43 = 101011_2)

The content behind the activity

A variety of strategies can be used to change a number into binary.

Example: To convert 43 to binary, start with the largest value on the game board (32). Ask, "Is 43 larger than 32?" The answer is yes, so place a red dot (representing 1) in the box under 32. Next, ask the same question for the next value (16). Compare 16 to the remainder of the original number minus the value just turned "on" (i.e., 43-32 = 11); "Is 11 greater than 16?" Since the answer is no, place a yellow dot to

Curriculum Standards:

Factors
(Common Core Math Standards: Grade 4, Operations & Algebraic Thinking, 4)

Number Sense and Place Value
(Common Core Math Standards: Number & Operations in Base Ten, Grade 4, 1; Grade 5, 1 & 2)

Exponents
(Common Core Math Standards: Grade 6, Expressions & Equations, 1)

Additional standards at: http://www.raft.net/raft-idea?isid=302

The content behind the activity (continued)

indicate that 16 is "off". Next would be, "Is 11 greater than 8?" The answer is "yes", so 8 must be "on", and a red dot is placed. This continues until all values have been checked.

Emphasize the connection between binary and decimal numbers. For example: the decimal number 36 is equal to three 10's plus six 1's. Similarly, its binary value (100100) is equal to one 32 plus one 4. Just as the place values in the decimal representation are powers of 10, the place values in the binary representation are powers of 2.

Application to Computing

Ones and zeros, also called "bits", are not explicitly written on the hard drive of a computer. In reality, they are represented by the orientation of magnetic particles on the disk. Since bits by themselves represent small amounts of information, they are commonly stored together in groups of 8, called "bytes".

Learn more

- Practice going the other way: convert binary to decimal.
- Investigate interesting characteristics of binary numbers. For instance, to multiply a binary number by 2, simply add on another 0 in the least significant (rightmost) bit.
- Extend this activity by creating a grid for another base system (such as base-5). Practice writing numbers in that system. Compared to base-2, does base-5 take more or fewer digits to represent a given number?
- Make and use a new grid going up to 2 to the 10^{th} power.

Related activities: See RAFT Idea Sheets:

Binary Weaving -
http://www.raft.net/ideas/Binary Weaving.pdf

Binary Birthday Bracelets -
http://www.raft.net/ideas/Binary Birthday Bracelets.pdf

Binary Bracelets for the 4th of July -
http://www.raft.net/ideas/Binary Bracelets for the 4th of July.pdf

Gold Bar Puzzle -
http://www.raft.net/ideas/Gold Bar Puzzle.pdf

Resources

Visit www.raft.net/raft-idea?isid=302 for "how-to" video demos & more ideas!

- **Connecting Binary Place Value & pattern recognition** – http://www.mathmaniacs.org/lessons/01-binary/
- **Converting between Binary and Base Ten number systems** – http://www.purplemath.com/modules/numbbase.htm
- **Information on binary numbers from the Khan Academy** – https://www.khanacademy.org/math/pre-algebra/applying-math-reasoning-topic/alternate-number-bases/v/number-systems-introduction

Binary Dots – Gameboard Template

32	16	8	4	2	1

32	16	8	4	2	1