Explore Star Polygons

See RAFT Idea Sheet ***Advanced Mathematical Dream Catchers*** - http://www.raft.net/ideas/Advanced Mathematical Dream Catchers.pdf for an introduction to Star Polygons.

Regular Star Polygons

For each value of ***n***, only a finite number of different regular star polygons can be constructed.

For each value of ***n*** shown below, compute the degree measure of each of the central angles which was used to locate the ***n*** equally spaced points on the circle. Record the possible values for ***r***, remembering that ***r*** must be less than ***n***, and that ***n*** and ***r*** must be relatively prime.

Record in the table the number of different regular star polygons that can be constructed for each value of ***n***. The case for ***n*** = 5 is shown as an example:

The number of equally spaced points *n*	**The degree measure of each central angle; $360^o \div n$**	**Possible values for *r* such that *r* < *n* and *r* & *n* are relatively prime**	**The number of different Star Polygons**
5	72°	1,2,3,4	2
6			
7			
8			
9			
10			
11			
12			
13			
REGULAR STAR POLYGONS			

Answer the following based on the chart above:

1. If ***n*** is a prime number, find a formula that will give the number of different regular star polygons for ***n***:

2. If ***n*** is not a prime number, describe a quick way to find the number of different regular star polygons for ***n***:

3. In modular arithmetic, for ***n***, $1 + (n - 1) = n = 0$
   $2 + (n - 2) = n = 0$
   $3 + (n - 3) = n = 0$
   These pairs of numbers, that have a sum of 0, are called additive inverses

Find the pair of additive inverses for the following:

a) Mod 5 : (0,0) (1,4) (2,3)
b) Mod 6 : (0,0) (1,5) (2,4) (3,3)
c) Mod 7 :
d) Mod 8 :
e) Mod 9 :
f) Mod 10 :
g) Mod 11 :
h) Mod 12 :
i) Mod 13 :

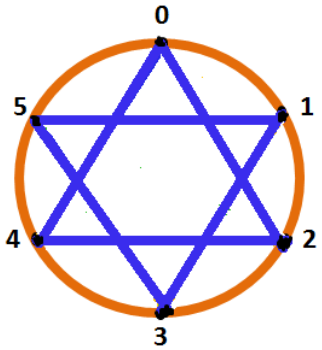
What can you say about regular star polygons for additive inverses $\{n \mid r\}$ and $\{n \mid n - r\}$:

Explore Modified Star Polygons!

Now let's consider the designs produced when ***n*** and ***r*** are not relatively prime! We continue with the notation {***n*** | ***r***} to represent these modified star polygons.

Look at this example→

Make a modified star polygon {6 | 2} (NOTE: ***n*** = 6 and ***r*** = 2)
First draw 6 equally spaced points on the circumference of the circle. Label the points from 0 to 5 in a clockwise order. Starting at 0 connect every second point in the sequence 0-2-4-0. Then select a point that has not been connected, say at point 1. Starting at 1, connect every second point in the sequence 1-3-5-1. Each of the 6 points has now been touched and the modified star polygon looks like:



Make a modified star polygon {6 | 3} (NOTE: ***n*** = 6 and ***r*** = 3)
In this example, starting at 0, you skip to the third point, which is 3, and then back three points to point 0, in the sequence 0-3-0. Likewise, starting at point 1 and skipping 3 points, you get the sequence 1-4-1. Then the last sequence of points is 2-5-2 (see Figure 1 below).
Figure 2 below shows double modified star polygons {6 | 2} and {6 | 3}.

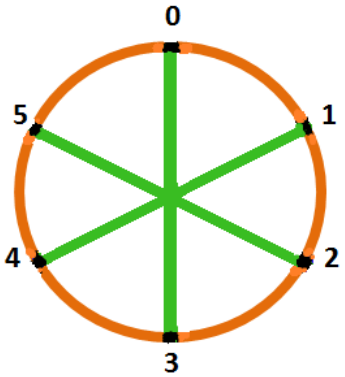


Figure 1

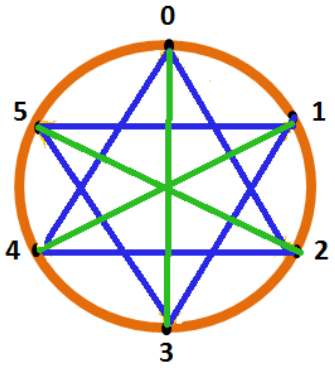


Figure 2

(When you combine 2 regular star polygons, 2 modified star polygons, or one of each kind the result is called a double star polygon) → written above as {6 | 2, 3}
For each value of ***n***, only a finite number of different modified star polygons can be constructed.

Let's explore a certain number of different modified star polygons that can be made for certain values of ***n***. A pattern will emerge, and then we can extend that pattern to larger values of ***n***.

For each value of ***n*** shown below, record the possible values of ***r***, (remember that ***r*** must be less than ***n***, and ***r*** and ***n*** must share a common factor other than 1). Then record in the table below the number of different modified star polygons that can be made for each value of ***n***. The example for ***n*** = 6 has been done already.

The number of equally spaced points *n*	**Possible values for *r* such that *r* < *n* and *r* & *n* are <u>not</u> relatively prime**	**The number of different modified Star Polygons**
6	2,3,4	2
8		
9		
10		
12		
25		
MODIFIED STAR POLYGONS		