Explore Modular Multiplication & Circle Designs

Beautiful designs result when equally spaced points on a circle are connected in a sequence based on a modular multiplication pattern. These designs are called "product designs".

1. First, create a sequence of numbers which will become the pattern for connecting points on the circle.
2. Choose a particular modulo, ***m***, and a positive integer ***p*** with ***p*** between 1 and ***m.***

3. Take ***p*** as the first number in the sequence, and generate successive numbers by multiplying repeatedly by ***p***, using mod m multiplication.

4. End the sequence when you reach ***p***, the starting point.

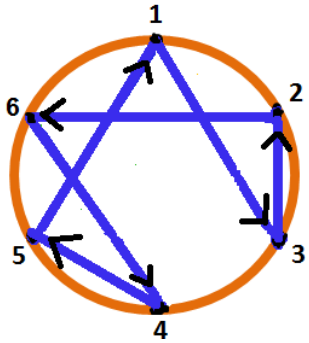
<u>Look at the following examples</u>→

<u>Example 1</u>: Let ***m*** = 7 and ***p*** = 3. Construct a product design based on this number sequence.

First, start with the number 3, the first number in the sequence. Find successive numbers by multiplying repeatedly by 3, using mod 7 multiplication (to multiply in mod 7, use this method: multiply in the ordinary way and divide the product by 7; the remainder will be the product in mod 7). Using this information, we get:

	p = 3
3 x 3 =	9 = 2 (mod7)
3 x 2 =	6 = 6 (mod7)
3 x 6 =	18 = 4 (mod7)
3 x 4 =	12 = 5 (mod7)
3 x 5 =	15 = 1 (mod7)
3 x 1 =	3 = 3 (mod7)

The sequence that is generated with ***m*** = 7 and ***p*** = 3 is: 3,2,6,4,5,1,3
These numbers include every number from 1 to ***m*** – 1 (which is from 1 to 6)
To create a dream catcher design based on this sequence, use a circle with ***m*** – 1 equally spaced points, and label the points from 1 to ***m*** – 1 (1 to 6), then connect the points with string in the sequence 3,2,6,4,5,1,3 which results in the product design {7 | 3} shown below:



Find a line of symmetry here

Product designs can be described by this notation: {***m*** | ***p***}, where ***m*** is the modulo and ***p*** is the constant multiplier.

Example 2: Construct the product design {7 | 2}.

Start with p = 2 and generate a sequence by multiplying repeatedly by 2, using mod 7 multiplication. This is how the sequence is created:

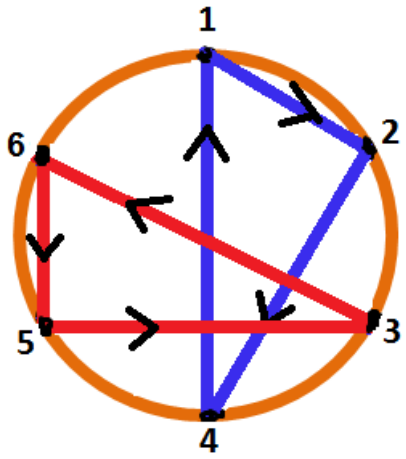
p = 2
2 x 2 = 4 = 4 (mod7)
2 x 4 = 8 = 1 (mod7)
2 x 1 = 2 = 2 (mod7)

The sequence we generate with ***m*** = 7 and ***p*** = 2 is 2,4,1,2

NOTE: since the sequence of the products 2,4,1,2 reaches the starting point number 2 without including all the points from 1 to 6, you then select one of the unused numbers, say 3, and use it as the starting number for another sequence. Again multiplying repeatedly by 2, you will generate the second sequence:

(starting number 3)
2 x 3 = 6 = 6 (mod7)
2 x 6 = 12 = 5 (mod 7)
2 x 5 = 10 = 3 (mod 7)

The second sequence starting at point 3 with ***m*** = 7 and ***p*** = 2 is 3, 6, 5, 3
This sequence combined with the first sequence includes all the numbers from 1 to 6. See the design below, with the first sequence 2-4-1-2 in red, and the second sequence 3-6-5-3 in blue.



Find a line of symmetry in this product design

Related activities: See RAFT Idea Sheets:

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Scalloped Circle String Art –
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